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Problem solving 1

Graphs and Groups, Spectra and Symmetries
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Let $L = SL_3(2) \cong PSL_3(2)$ and let V_3 be the natural module of L, so that V_3 is a 3-dimensional $GF(2)$-space of which L is the automorphism group, and let V_1 be the trivial 1-dimensional $GF(2)$-module of L. Let $0 \neq v \in V_3$. Denote by H the stabilizer of v in L.

(1) Calculate the number of subgroups isomorphic to L in the semidirect product of V_3 and L with respect to the natural action of L on V_3.

(2) Show that there is the unique indecomposable extension of V_3 by V_1 (that is a 4-dimensional $GF(2)-$module whose only proper submodules are isomorphic to V_3 and the unique module $V_1 \setminus V_3$).

(3) Is there an uniserial module $V_1 \setminus V_3 \setminus V_1$?

(4) What is the submodule structure of

(a) the $GF(2)-$permutation module of L on the cosets of subgroups H, H', $F = F(7,3)$ (which is a Frobenius group with the core of order 7 and a complement of order 3), F' (which is a cyclic group of order 7)?

(b) $V_3 \otimes V_3$, $V_3 \otimes V_3^*$ (where V_3^* is the dual of V_3)?

(5) Let $W = V_3^* \setminus V_3$ be the direct summand of the permutational module of L on the cosets of H. Let X be the semidirect product $W \rightthreetimes L$. Calculate the number of subgroups of X which are isomorphic to L.